V.C. An Alternative Approach to Cross Impact Analysis

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Abstract

This paper presents the theoretical justification for the use of a particular analytical relation for calculating inferences from answers to cross impact questions. The similarity of the results to other types of analogous applications (i.e., logic regression, logistic models, and the Fermi-Dirac distribution) is indicated.

An example of a cross impact analysis in an interactive computer mode is presented. Also discussed is the potential utilization of cross impact as: (1) A modeling tool for the analyst, (2) A consistency analysis tool for the decision maker, (3) A methodology for incorporating policy dependencies in large scale simulations, (4) A structured Delphi Conference for group analysis and discussion' efforts and (S) A component of a lateral and adaptive management information system.

He took the wheel in a lashing roaring
    hurricane
And by what compass did he steer the course
    of the ship?
"My policy is to have no policy," he said in
    the early months,
And three years later, "I have been controlled
    by events."

"The People, Yes"
Carl Sandburg

Introduction

In Delphi[1] design, one of the major problems has been how to obtain meaningful, quantitative subjective estimates of the respondents' individual view of causal relationships among possible future events. Meaningful, in this context, is the ability to compare the quantitative estimates of one respondent with those of another respondent

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[1] See "The Design of a Policy Delphi" by Murray Turoff, in *Technological Forecasting and Social Change*, *2*, No. 2 (1970), for an explanation of the Delphi technique and a comprehensive bibliography.

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and correctly infer where they differ or agree about the amount of impact one event may have on another.

A number of design techniques, or question formats, have evolved as approaches to this problem. The particular formalism which has received comparatively wide usage, due to the ease of obtaining answers to a fairly involved problem, is the "cross impact" question format first proposed in a paper by Gordon and Hayward (see bibliography). However, the analytical treatments proposed as methods of either checking consistency or drawing inferences are essentially heuristic in nature and exhibit various difficulties. The Monte Carlo approach, which is in widest use, is particularly unsuited to obtaining a consistent set of estimates through individual modification. This is because the assumptions upon which the Gordon (Monte Carlo) approach is based imply inconsistency in the estimates provided. The analytical approach described in this paper was developed specifically for restructuring the cross impact formalism in a manner suitable for use on an interactive computer terminal. This requires that the user be able to modify or iterate on his estimates until he feels the conclusions inferred from his estimates are consistent with his views.

The type of event one is usually considering in the cross impact formalism may not occur at all in the time interval under consideration. Furthermore, an event may be unique in that it can only happen once. Examples of the latter are:

The development of a *particular* new product
The occurrence of a *particular* scientific discovery
The passage of a *specific* piece of legislation
The outbreak of a *particular* war.

For this type of event there is usually no statistically significant history of occurrence which would allow the inference of a probability of occurrence. While there are sometimes historical trends for certain general items, such as the overall occurrence rate of scientific discoveries, specific scientific discoveries may fall outside this general trend. The probability, of a cure for a particular type of cancer is one example. In this instance, one would make his estimate dependent upon a rather significant number of other events, involving such factors as the success of non-success of a large number of specific research projects that may provide information only on the nature of the disease and thereby influence in some manner the discovery *of* a cure. Also, of course, one would consider events related to the provision of funds necessary to start research projects in areas that should be, but currently are not, explored. This latter consideration may lead to the enumeration of additional events related to the economic and political environment determining the availability of funds.

It is quickly realized from the above example that the first step in the construction of a cross impact exercise is the problem of specifying the event set. At present, one workable and popular approach to this problem is to allow the individuals who will participate in the cross impact exercise to specify the set of events they feel are crucial to the problem under consideration. This process may be conducted in a face-to-face conference or committee approach, or in a Delphi exercise. The success of the exercises, in terms of specifying a good event set, is dependent upon the knowledge the group has about the problem, as is the value of the quantitative estimates that will be obtained. However, there are some situations where the formalism may be used as an educational tool on some groups to expose the complexities of the problem. This usually occurs when a non-expert group evaluates an event set generated by an expert group and the evaluation by the expert group is available for comparison.

Once an event set is specified, the first step is to ask a person what he estimates is the chance of that event occurring in the interval of time from now to some point in the future (i.e., ten years). An individual viewing an event dependent upon causal effects normally has a very discontinuous view of the event happening over time. If, for example, an event specified in the cross impact set is the expectation of receiving a. raise in excess of a certain amount within the next year, then the individual concerned may feel that the event, from a time-dependent view, can only occur at certain sub-intervals of the year. The raise may normally only be possible at the completion of the mid-year and year-end reviews. To answer the question as stated, he must perform some averaging process taking into account the time dependence as well as causal effects from other potential occurrences.

Assume that one of the other events is the receipt of a letter from the head of the organization by his supervisor expressing a great deal of satisfaction with the results of the project this individual is currently undertaking. This can only occur after the project is completed and over that interval which is normal for review of the project. Let us hypothesize that the individual has a pessimistic estimate of this occurring. His view on these events and others in this particular event set represents his view or opinion of the world concerned with his getting a raise and is the first step in the cross impact procedure.

The next step in the cross-impact procedure is to perturb his view of the world (or to create a new world) by telling him to assume a certainty that one of the events will (or will not) occur and asking him to reconsider other events. In the example, assume that we tell him it is certain that the head of the organization will send a letter to his supervisor expressing satisfaction with the results of the project. This may cause him to re-evaluate upward his expectancy for getting the raise. More important for understanding the cross impact formalism, this may cause him to arrive at a completely new time dependency for the probability of getting a raise. In other words, a time interval during which he thought there was no probability of getting a raise because it occurred between the mid-term and year-end review could not become a very probable time interval for getting the raise because it occurs after the project is completed.

The important point to recognize now is that if we had extracted all the information contained in the time-dependent view of this event set, we could have used some of the standard relationships in probability theory to check the consistency of estimates at *each* point in time for *each* world view. This, as will become evident in the rest of the paper, would be an infeasible amount of information to ask an individual to provide for more than a few events. Rather, the cross-impact problem is to infer the causal relationships from some relationship among the different world views established by perturbing the participant's initial view with assumed certain knowledge as to the outcome of individual events. These different world views may represent, at least implicitly, different time-dependent distributions for the same event. Therefore, the probability estimates for the occurrence of each event, resulting from some subjective averaging procedure over the total time interval, do not conform to the definition of a unique probability space in the classical sense, to which the standard relationships between such concepts as prior and posterior probabilities may be applied. Rather, we are asking if there is some model or relationship, based upon causal effects, which can be used to relate a number of separate probability spaces.

We are faced with a situation analogous to some degree with the problem in quantum mechanics where, in order to measure the state of the system we must physically disturb it. In this case, in the process of setting up an instrument to measure the estimates of an individual's view of causal relationships we disturb those estimates. The concept of defining a measuring instrument is crucial, since in most cross impact exercises we wish to be able to compare estimates among different individuals. Unfortunately, unlike quantum mechanics, any analogy to a Planck's constant may differ from individual to individual and therefore we do not have available an analogous uncertainty relation.

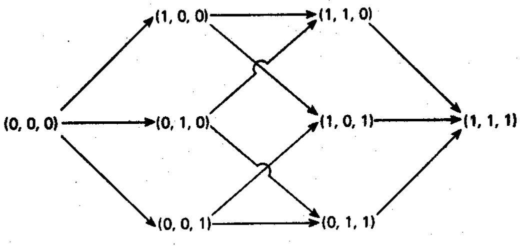
The THEORY section of this paper attempts to describe and justify the analytical procedures for such a measurement device. It assumes that the reader has some familiarity with the literature on cross impact and the other analytical approaches which have been proposed for handling this problem. If this is not the case, the reader should perhaps read the EXAMPLE and APPLICATIONS section prior to reading other parts of this paper.

Theory

Structure of *the Problem*

Events to be utilized in a cross impact analysis are defined by two properties. One, they: are expected to happen only once in the interval of time under consideration (i.e., nonrecurrent events) and two, they do not have to happen at all (i.e., transient events). If one holds to a classical "frequency" definition of probability than it is, of course, pointless to talk about the probability of a nonrecurrent event. We, therefore, assume an acceptance of the concept of a subjective probability estimate having meaning for nonrecurrent events. When dealing with recurrent events within the cross-impact framework, they should be restated as nonrecurrent events by either specifying an *exact* number of occurrences within the time interval or utilizing phrases such as "... will happen at least once." Any recurrent event may be restated as a set of nonrecurrent events.

If we are considering N nonrecurrent events in the cross impact exercise there are then 2^N distinct outcomes spanning the range from the state where none of the events have occurred to the state where all of them have occurred. If *we* are in a state where a -particular set of K of the events have occurred, then there are at most $N - K$ remaining possible transitions to those states where $K + 1$ events have occurred. Since it is possible that no additional event will occur, the sum over these $N - K$ transition probabilities need not add to one. The amount by which the sum is less than one is just the probability that the system remains in that particular state until the end of the *time* interval. Once the 'system has moved out of a particular state, it will never return to it since each event is assumed to occur once and only once. The total number of possible transition paths and equivalent transitions probabilities (allowable paths) needed to *specify this* system of 2^N states is $N2^{N-1}$.[2] An example of all states and transitions for a three event set is diagrammed below, where *a* zero denotes nonoccurrence and a one denotes occurrence of the event. The events are, of course, distinguishable and it is also assumed two events do not occur simultaneously.



Transitions and states for three events (1, 2, 3).

One can see that as *N* gets larger than three it quickly becomes infeasible to ask an individual to supply estimates for all the transition probabilities. The cross impact formalism, as an alternative, has had widespread usage because: one, it limits itself to N^2 questions[3] for N events; and two, the type of question asked appears to parallel the intuitive reasoning by which many individuals view "causal" relationships among events. However, it does pose a serious theoretical difficulty for extracting or inferring conclusions based upon the estimates supplied, since the answers supplied are both insufficient and different information fom that required to completely specify the situation. This is easily seen by relating the answers to the cross impact questions in terms of the original transition probabilities. The first cross impact question which is asked for all *N* events (i = 1 to *N) is*

(1) "What is the probability that an event, i, occurs before some specified future point in time?"

The answer to this can be related to the appropriate transition probability sum taken over all independent paths leading to all states in which event i occurs (i.e., one-half of the states). However, when the second cross impact question is asked for the remaining (N- 1) events relative to a j-th event:

(2) "What is your answer to question (1) if you assume that it is certain to all concerned that event j will occur before the specified point in time?"

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[2] Assuming the system transition probabilities are independent of past history. The history or memory dependent case is discussed later.

[3] $N2^n$-^{1} is the number of questions one would have to ask to obtain quantitative estimates to completely specify the model.

We have in effect altered the original set of transition probabilities. This latter question is equivalent to imposing a set of constraints upon the transition probability estimates of the following form:

The sum over all the transition probabilities leaving a state in which the j-th event has not occurred must be equal to one.

The above must be the case since we cannot remain or terminate in a state for which the event j has not occurred. What we have done, at least subconsciously, to the estimator is to ask him, in the light of new constraints, to create a whole new set of prior transition probabilities. This creates an analytical problem in trying to relate the original transition probabilities estimated under the constraints. One may consider this as a problem in trying to relate different "world" views:

The so-called "conditional" probabilities derived from the second "cross impact" question are not the conditional probabilities defined in formal probability theory. Rather the answer to the second cross impact question might better be termed as a "causal" probability from which one would like to derive a "correlation coefficient" which provides a relative measure of the degree of causal impact one event has upon another. However, the term "conditional probability" has become so common in a lay sense that it is often easier to communicate and obtain estimates by referring to the answers to the second cross impact question as "conditional" probabilities.

The previous points may be illustrated by the following examples where the reasonable answers, to the cross impact questions, do not obey the mathematical requirements associated with standard conditionals or posterior probabilities. The first example is a "real" illustration and the' second is an abstract urn representation of what is taking place. Consider the following two potential events:

Event 1

Congress passes a strict and severe law specifically restricting mercury pollution by 1975.

Assume the probability estimate of occurrence is e_1:

$$P(1) = e_1$$

Event 2

At least 5,000 deaths are directly attributed to mercury pollution by 1975. Assume the probability estimate of occurrence is e_2:

$$P(2) = e_2.$$

If it is certain that Congress will pass the above law by 1975, either Event 2 is not affected or its probability may decrease if the law is enacted soon enough to reduce levels of pollution before 1975. Therefore, the probability of Event 2 given that Event 1 is certain should be less than or equal to the original estimate,

$$P(2{:}1) = e_3 = e_2.$$

If it is certain that five-thousand people or more will die (Event 2) by 1975, then most rational estimators will increase their estimate of the probability for Congress passing the law,

$$P(1{:}\,2) = e_1 + ? \text{ where } ? > 0 \text{ and } e_1 + ? = 1.$$

If $P(2{:}1)$ and $P(1{:}2)$ were standard conditionals (i.e., posterior probabilities), we would conclude that the probability of both occurring is

$$P(1, 2) = P(1{:}2)P(2) = P(2{:}1)P(1).$$

However,

$$P(1{:}2)P(2) = e_1\, e_2 + ?\, e_2,$$
$$P(2{:}1)P(1) = e_3 e_1 = e_2 e_1 < P(1{:}2)P(2).$$

Therefore, $P(1{:}2)$ and $P(2{:}1)$ are not the standard conditional probabilities.

A valid theoretical point may be made by arguing that the above problem would be eliminated by designing an event set consisting of mutually exclusive events as a basis vector from which a decision tree or table can be constructed and to which can be applied a Bayesian type analysis. However, in practice, economic, political, and sociological types of questions, often examined in the cross impact scheme, do not lend themselves to defining such a set, and, if they do, the number of events which have to be considered may grow too large for the purpose of obtaining estimates.

As the second example consider two urns, labeled urn one and urn two, in which are distributed a large number of black and white balls. An individual who is to estimate the chance of drawing a white ball from either urn has available two pieces of information:

(1) Two-thirds of the balls are white.
(2) Urn number two always contains at least one-quarter of the balls.

Given no other information, if the estimator were asked the probability of drawing a white ball from urn two (or urn one) his best estimate is two-thirds.

Now the estimator is supplied with a new piece of information:

(3) The probability of drawing a white ball from urn two is zero.

Then the estimator can infer from (2) and (3) that at least one-quarter of the balls, all of them black, are in urn two. From this and (1) the probability of drawing white ball from urn one lies between one (assuming all black balls are in urn two) and eight-ninths.[4] Assuming the distribution of probabilities between one and eight-ninths is uniform (no other information) the best estimate for the probability of drawing a white ball from urn one is half way between one and eight-ninths, or 17/18.

Suppose, on the other hand, instead of item three we supply the estimator with the following information:

(4) The probability of drawing a white ball from urn one is zero.

Now he knows that between none and all the black balls can be in urn one. This means the probability of drawing a white ball from urn two is between two-thirds and one. The midpoint estimate in this case is five-sixths.

These resulting four estimates are summarized in the following table:

Probability of drawing a white ball from	based upon (1) and (2) only	given (3) in addition to (1) and (2)	given (4) in addition to (1) and (2)
urn one	2/3	17/18	0
urn two	2/3	0	5/6

Note that, in this example, we have never drawn a ball from urn one or two; therefore, there is no posterior probability provided. The two probabilities calculated by assuming the information items (3) and (4) are new priors based upon assumed knowledge as to the state of the system.

The cross impact analysis problem in terms of the example is: Given the four estimates made in the table to what extent can the information items (1) and (2) be inferred analytically. In other words, will the relationships derived from the estimates provide a description of this system which behaves similarly or approximately like the system described by knowing explicitly items (1) and (2). The goal of the cross impact in this example would be to infer a model, from the four estimates provided, which will allow a prediction of the probability estimate of drawing a white ball from urn one if the estimator is given explicitly the probability of drawing a white ball from urn two, or vice versa. In essence, we wish to create an analytical model of his knowledge about the situation.

Another view of cross impact is to consider it as an attempt to obtain subjective estimates of correlation coefficients. Gordon's approach to this problem asks directly for these coefficients, while the approach in this paper is to ask for probabilities from which the correlation coefficients can be calculated. The transition from formal probabilities to subjective probabilities, or likelihood estimates, is not difficult to make. However, the formal theory of correlation coefficients in statistics does not specify a unique analytical definition of a correlation coefficient in the same sense that a unique

[4] Assuming urn two contains only ¼ of all the balls, this leaves $\frac{1}{12}=(\frac{1}{3}-\frac{1}{2})$ of the black balls in urn one. Then $\frac{8}{9}=\frac{2}{3}\Big/(\frac{2}{3}+\frac{1}{12})$.

measure of probability is defined. Therefore, the problem of defining subjective estimates of correlation coefficients to measure causal impact (whether direct, as in the Gordon approach, or indirect, as this approach) rests on a more intuitive foundation than does the concept of subjective probability.

The separate justifications presented on the following pages for a particular approach to the cross impact problem are all heuristic in nature. Since we are trying with N^2 items of information to analyze a problem requiring $N2^{N-1}$ items *of* information for a complete solution, it would, therefore, seem that any approach to the analysis *of* the problem is an approximation. Also, there does not appear to be any explicit test which will judge, one approach to be better than another. One significant measure of utility is the ease with which estimators can supply estimates and whether they feel the consequences inferred by the approach from their estimates adequately represent *their* view of the world.[5] The author feels that the method developed in this paper *offers* the estimator a greater opportunity to arrive at a consistent set of estimates and inferences than is available to him in the techniques currently reported in the literature on cross impact. The mathematical relationship developed is not new; it has been *used* in physics, statistics, operations research, and information theory in modeling situations where one *is* concerned with the probability of the outcome of random variables which can only take on zero or one values (see annotated bibliography) However, in many such cases one is dealing with a recurrent process where the model *can* be experimentally verified in at least some sense.

It should be noted that the state transition model of the interaction among events, which we have adopted to illustrate conceptually the meaning of the cross impact questions, provides only a lower bound $(N2^{N-1})$ on the number of parameters needed *to* completely specify the problem. It inherently assumes that once a given state (i.e., a specified set of events has occurred) is attained, the determination of the transition probabilities that leave that state (going to a state where one more event has occurred) is independent of the path used to move from the zero state (i.e., no events have occurred) to the state under examination (i.e., a system without memory[6]). If we had assumed the possibility of a completely different set of transition probabilities out of the given state, each set dependent upon the path that might have been used to arrive at the state, then the number of transition probabilities needed to completely specify the total problem would be:

$$\sum_{j=1}^{N} j! \binom{N}{j} \cong eN! \quad \text{while} \quad N \to \infty.$$

The real world, as modeled by a particular event set, is probably a mix of memory and non-memory dependent situations. Therefore the number of parameters one would theoretically require to specify all the information falls between the two limits. The

[5] This is not to say the estimator's view of the world may not be wrong, but that it may be overly presumptuous to expect the model to be able to correct the estimator's view. This is contrary to some cross impact approaches.

[6] Implies the system can be modeled as a Markov Chain.

following table contrasts the data demands *of* the cross impact analysis with the memory and no memory limits.

Number of Events	Cross Impact	No Memory System	Memory System
N	N^2	$N2^{N-1}$	$\approx eN!$
2	4	4	4
3	9	12	15
4	16	32	64
5	25	80	325
10	100	5120	$\approx$ 10 million

Since most cross impact exercises deal with a range of 10 to 100 events, it is fairly obvious why no attempt is made to obtain estimates which would completely specify the problem.

The basic interpretation of cross impact conditionals as a new set of prior probabilities is not affected by the issue of whether or not one is dealing with a system that has a memory. This issue does arise, however, when one tries to describe or model the question of time dependence. This subject has not been adequately addressed in reported attempts to modify the cross impact formalism to allow variation of the time interval for the purpose of arriving at an explicit time dependent model.

In summary then, the cross impact approach in its most general context is an attempt to arrive at meaningful analyses of a system composed of transient, nonrecurrent events which may or may not be dependent upon history (i.e., memory). It is, however, fairly obvious that with event sets of the order of ten in size we have arrived at a point where it is desirable to find some sort of macro or statistical view of the problem as opposed to any attempt at enumerating all micro relationships such as the transition probabilities for all paths. This is analogous to the choice of trying to write dynamic equations for each particle in a gas or to utilize a set of relations governing the collective behavior of the gas.

The following five sections contain a number of alternative methods for arriving at the mathematical relationship used to model the cross impact problem. The explicit use of the resulting relationship for obtaining estimates from an individual is described in the EXAMPLE section of the paper. All the derivations provided are heuristic; however, the last two represent a fairly formal approach and provide some insight to the exact nature of the approximations being made.

Difference Equation

Given a set of events which may or may not occur over an interval of time, we assume that an individual asked to estimate the probability of the occurrence of each event will supply a "consistent" set of estimates. In other words, his estimate for the probability of the i-th event (out of a set of N events) includes a subjective assessment of the other events in terms of their probability of occurrence over the time frame and any "causal" relationships they may have upon one another. Under this assumption we may hypothesize that there exists a set of *N* equations expressing each of the probabilities (P, for i = 1 to N) as a function of the other N- 1 probabilities:

$$P_i = P_i(P_1, P_2, \ldots, P_k, \ldots, P_N) \quad \text{for} \quad k \neq i. \tag{1}$$

The above functional may also include other variables expressing the causal impact of potential events not specified in the specific set of *N* events.

If the individual making the estimate receives new information which would require a change in his estimates for any of the probabilities, then his changes should be consistent with the difference equation form of (1):

$$\delta P_i = \sum_{k \neq i} \frac{\partial P_i}{\partial P_k} \delta P_k + \frac{\partial P_i}{\partial \beta} \delta \beta, \tag{2}$$

where ß is considered to be a collective measure of the impact of those events not included in the specified set. This will become clearer in later sections.

The boundary condition that must be satisfied by each of these equations is that if an event is certain to occur or certain to not occur then no change in the environment (as represented by the other events) can influence the outcome of the "certain" event, i.e., if

$$P_i = 1 \quad \text{or} \quad 0, \qquad \text{then} \quad \delta P_i = 0. \tag{3}$$

The simplest (in an algebraic sense) manner in which this can be satisfied is to assume

$$\frac{\partial P_i}{\partial X} = P_i(1 - P_i) \frac{\partial G_i}{\partial X}, \tag{4}$$

where X is any of the variables of differentiation on the right side of (2) and G is an arbitrarv function. Therefore we may rewrite (2) as:

$$\delta P_i = P_i(1 - P_i) \left[\sum_{k \neq i} \frac{\partial G_i}{\partial P_k} \delta P_k + \frac{\partial G_i}{\partial \beta} \delta \beta \right]. \tag{5}$$

The next assumption is to consider the partial derivatives with respect to the P_k's as constants:

$$\frac{\partial G_i}{\partial P_k} = C_{ik}. \tag{6}$$

The whole string of assumptions to this point is based upon an appeal to simplicity.[7] We may now solve (5) as a differential equation to obtain

$$P_i = \frac{1}{1 + \exp\left(-\gamma_i - \sum_{k \neq i} C_{ik} P_k\right)}, \tag{7}$$

where the ?[1] may be a function of unknown variables 'ß' and also incorporates a constant of integration. One may easily verify that Eq. (7) is a solution to (5) by taking the total derivative.

This equation is recognizable as either the logistic equation which is often encountered in operations research or as a Fermi-Dirac distribution in physics. The implications of this will be discussed in later sections of this paper. The major difference in the assumptions leading to this result, as opposed to the Monte Carlo treatment of the cross impact problem developed' by Gordon and others, is the crucial assumption that the hypothetical estimator of the occurrence probabilities is consistent in his estimates. In practice, an individual asked to estimate a significant number of related quantitative parameters is unlikely to be consistent on the first attempt. There must therefore be a feedback process for the individual in order to allow him to arrive at what can be viewed as a consistent set of values. In the Monte Carlo approach it is impossible for the individual to reasonably determine from the results of the calculations whether an inconsistent outcome (with his view) is merely a problem in his juggling of a large set of numbers or a basic inconsistency in his view of causal relationships. The primary advantage of Eq. (7) therefore is to provide an explicit functional relationship which presupposes consistency and thereby provides the estimator the opportunity to arrive at consistency if he is provided with adequate feedback and opportunity to modify his estimates.

Underlying this view is the premise that the estimator would have a computer terminal available to exhibit the consequences of his estimates in terms of perturbations about the solution he initially provided. This then allows the estimator to determine if the resulting model adequately reflects his world view and to adjust his inputs accordingly. The lack of the ability for the individual estimator to first establish consistency for his own estimates is a major shortcoming in the current attempts to average in some manner the estimates of a group, as normally takes place in the cross impact Delphi exercises.

Likelihood Measure

Consider the following three measures which may be applied to the question of expressing the likelihood of the occurrence of a particular event (i.e., the i-th).

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[7] There is no merit in attempting complex models for processes until the limits of validity for the simplest models are understood. Some of these limits will be discussed in a later section of the paper.

Probability: P_i
Odds: $O_i = P_i / (1-P_i)$, and
Occurrence ratio: $F_i = F(P_i) = \ln O_i = \text{In}\ [P_i/(1 - P_i)]$.

The boundary properties of these are summarized as follows:

	P_i	O_i	F_i
Event certain to occur	1	8	8
Random occurrence (i.e., neutral point)	½	1	0
Event certain to not occur	0	0	-8

All the above measures are to be found in the literature of statistics. The "occurrence ratio" is commonly referred to as the "weight of evidence" when applied to two different, but mutually exclusive, events. It has the interesting property of being anti-symmetric about the neutral occurrence point. In other words, given two estimates of the occurrence: P_i and P_i^* then if

$$P_i + P_i^* = 1 \quad \text{or} \quad (P_i - \tfrac{1}{2}) = -(P_i^* - \tfrac{1}{2}), \tag{8}$$

we have

$$\phi(P_i) = -\phi_i(P_i^*). \tag{9}$$

If we have a causal view with respect to the occurrence of an event, then we assume that the occurrence can be influenced by some policy, investment, or other type of "effort" directed at enhancing or inhibiting it. We would like, therefore, to establish some relationship between likelihood for the occurrence of the event and the effort invested in either promoting or preventing it (i.e., a negative or positive effort). We would also like this relationship to be such that if an equal amount of effort is devoted to both enhancing and preventing the occurrence of the event then the likelihood corresponds to a probability of one-half (i.e., random or neutral). In terms of the measures of likelihood commonly employed, the only one which might be assumed to be directly proportional to a measure of effort is the occurrence ratio. Therefore we assume:

$$\phi(P_i) \propto E_i^{\tau}. \tag{10}$$

Where $E_i^?$ is the sum of all the effort invested in either bringing about the event (positive effort) or preventing it from occurring (negative effort).

Effort is the type of quantity which might be measured by the actual dollars invested in the goal. However, in many interesting cases we cannot model or measure the effort directly. We must, therefore, establish an empirical or indirect measure of effort. This can be done by assuming that the effort is measured by relating it to all other events which have a causal relationship to the i-th event:

$$E_i^\tau = \sum_{k \neq i} [D_{ik} P_k + F_{ik}(1 - P_k)]. \tag{11}$$

We may rewrite this, to correspond to our earlier notation, as

$$E_i^\tau = \gamma_i + \sum_{k \neq i}^{N} C_{ik} P_k, \tag{12}$$

where the ?$_i$ may also include the events which have already been determined with respect to their occurrence of nonoccurrence as well as the events we are not specifying in the set of N events. We then have

$$\phi(P_i) = \ln [P_i/(1 - P_i)] = \gamma_i + \sum_{k \neq i}^{N} C_{ik} P_k. \tag{13}$$

This is the same result we arrived at earlier in Eq. (7). We note that while the contribution of the k-th event to the i-th event is additive in terms of the occurrence ratio, it is multiplicative with respect to the odds:

$$O_i \propto \prod_{k \neq i} O_{ik}, \tag{14}$$

where

$$O_{ik} = \exp(\dot{C}_{ik} P_k).$$

Therefore any change in the probability of one of the events affecting event i changes the odds multiplicatively; i.e.,

$$O_{ij}(P_j + \delta P_j) = O_{ij}(P_j) O_{ij}(\delta P_j). \tag{15}$$

It should be observed that the conclusions expressed by Eqs. (14) and (15) could have been used as initial assumptions in deriving the cross impact relationship represented by Eq. (13). We also note a functional analogy between the odds in this problem and the partition function in quantum statistical mechanics.

Another aspect of relations (14) and (15) is that they satisfy a likelihood viewpoint of statistical inference in that the final odds may be written as the product of the initial odds times a "likelihood ratio."

Useful Relations

It is useful, at this point, to introduce some relationships involving the occurrence ratio which are needed to actually apply the results to obtaining estimates. If we assume an event (the j-th) becomes certain to, occur, then we may define

$$R_{ij} = P_i \quad \text{for} \quad P_j = 1, \tag{16}$$

which is equivalently

$$\ln \frac{R_{ij}}{1 - R_{ij}} = \gamma_i + \sum_{k \neq i, j} C_{ik} P_k + C_{ij}. \tag{17}$$

Subtracting Eq. (17) from Eq. (13) we have

$$C_{ij}(1 - P_j) = \ln [R_{ij}(1 - P_i)/P_i(1 - R_{ij})] \tag{18}$$

or

$$C_{ij} = \frac{1}{1 - P_j} [\phi(R_{ij}) - \phi(P_i)]. \tag{19}$$

Therefore, if we know P_i, P_j, and R_{ij} we may calculate C_{ij}. We note $C_{ij} = 0$ if $P_i = R_{ij}$.

Similarly if we assume an event j becomes certain to not occur we may define

$$S_{ij} = P_i \quad \text{for} \quad P_j = 0. \tag{20}$$

Applying the same technique we obtain

$$C_{ij} = \frac{1}{P_j} [\phi(P_i) - \phi(S_{ij})]. \tag{21}$$

Therefore, if know P_i, P_j, and S_{ij}, we may also calculate C_{ij}. Or by combining Eqs. (19) and (21) we have

$$C_{ij} = \phi(R_{ij}) - \phi(S_{ij}) \tag{22}$$

which may be used to calculate S or R given C and either R or S respectively. If we have obtained values for all the C's then we can calculate ?$_i$ by

$$\gamma_i = \phi(P_i) - \sum_{k \neq i}^{N} C_{ik} P_k. \tag{23}$$

This is in essence the normalization condition.

Eliminating C_{ij} from Eqs. (19) and (21) we have the following interesting relationship between P, R, and S:

$$\phi(P_i) = \phi(R_{ij}) P_j + \phi(S_{ij}) (1 - P_j). \tag{24}$$

This is plotted for some representative values on the following graphs. One may consider this last equation as a utility function relationship. In this instance we are not considering the utility of an event in terms of some winnings. Rather we are asking what is the utility of the j-th event to the occurrence of the i-th event. The occurrence ratio for the i-th event satisfies all the necessary properties of a utility function. One could have derived the` cross impact relations by assuming the above utility relation and the condition that

$$\partial\phi(P_i)/\partial P_j = P_i(1 - P_i) C_{ij} \tag{25}$$

in order to satisfy the boundary condition that the event j can have no utility for the event i when event i is already certain to occur or not occur. The C's, therefore, may be interpreted as marginal utility factors relating the' utility of the j-th event to the i-th event. In a sense then, an alternative view of this cross impact approach is an assumption of a constant normalized marginal utility of one event for another.

Maximizing Information Added

Assuming we know the probability (P_i) that the i-th event will occur over some time frame, we wish to obtain two other probability estimates:

R_{ij} The probability of the i-th event, given the j-th event is certain to occur.

S_{ij} The probability of the i-th event, given the j-th event is certain to not occur.
The added information, over and above knowing P_i is defined as

$$I(i|j) = R_{ij}\ln(R_{ij}/P_i) + S_{ij}\ln(S_{ij}/P_i) + (1-R_{ij})\ln\frac{1-R_{ij}}{1-P_i} + (1-S_{ij})\ln\frac{1-S_{ij}}{1-P_i}. \tag{26}$$

It should be noted that the nonoccurrence of the event i is also considered significant information, hence-the last two *terms in* the *above* equation. We also see

$$I(i|j) = 0 \quad \text{if} \quad S_{ij} = R_{ij} = P_i, \tag{27}$$

i.e., *no* added information.

We assume that if *the* values of R_{ij} and S_{ij} are correlated in any manner, then the correlation is such as to maximize the added information

$$\frac{\partial I(i|j)}{\partial R_{ij}}dR_{ij} + \frac{\partial I(i|j)}{\partial S_{ij}}dS_{ij} = 0, \tag{28}$$

which results in

$$dR_{ij}\ln[R_{ij}/(1-R_{ij})] + dS_{ij}\ln[S_{ij}/(1-S_{ij})] = (dR_{ij}+dS_{ij})\ln[P_i/(1-P_i)]. \tag{29}$$

Recalling that the occurrence ratio is

$$\phi(x) = \ln[x/(1-x)],$$

we may rewrite Eq. (29) as

$$\frac{dR_{ij}}{dS_{ij}}\phi(R_{ij}) + \phi(S_{ij}) = \left(\frac{dR_{ij}}{dS_{ij}} + 1\right)\phi(P_i) \tag{30}$$

or

$$\frac{dR_{ij}}{dS_{ij}} = \frac{\phi(P_i) - \phi(S_{ij})}{\phi(R_{ij}) - \phi(P_i)}. \tag{31}$$

The latter form indicates that *dR/dS* must always' be positive since if $P_i > R$ then $S > P_i$ or vice versa respectively.

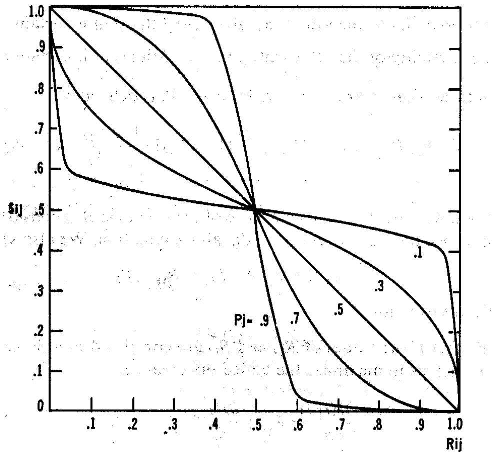
The necessary assumption to obtain our earlier results is that

$$\frac{dR_{ij}}{dS_{ij}} = \frac{P_j}{1 - P_j} = \text{Odds}(j). \tag{32}$$

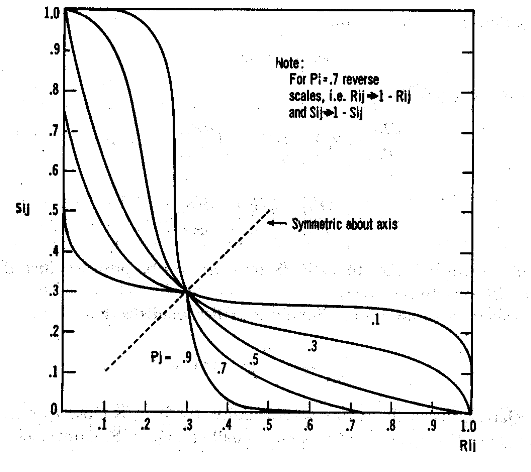
This behaves physically as one would desire, for if P_j is close *to* one, then a very large change in R is necessary to make a small change in S. Conversely if P_j is close to zero, a very large change in S is necessary to produce a small change in R. Also when $P_j = 1/2$ the relative change in R and S is equal.

R_{ij} VERSUS S_{ij}

For Pi = .5



For Pi = .3



This behavior is summarized in the following table where ? is a quantity close to zero and Eqs. (31) and (32) are linearized in ?.

If $P_j =$	ϵ	1/2	$1-\epsilon$
then $dR_{ij}/dS_{ij} =$	ϵ	1	$1/\epsilon$
and $P_i =$	$\frac{R\epsilon + S}{1+\epsilon} \cong S$	$\frac{R+S}{2}$	$\frac{R+\epsilon S}{1+\epsilon} \cong R$

So that as P_j approaches zero, P_i approaches S_{ij} and as P_j approaches 1, P_i approaches R_{ij}.

Substituting Eq. (32) into Eq. (30) or (31) we have

$$\phi(P_i) = \phi(R_{ij})P_j + \phi(S_{ij})(1 - P_j), \tag{33}$$

which is the earlier result, Eq. (24).

While this derivation is no less heuristic in nature than the previous sections, it does provide a fairly, explicit statement of only two assumptions, i.e., Eqs. (28) and (32), necessary to obtain the cross impact relations developed in this paper.

A Mixed Statistical Mechanics and Information Theory Approach

Consider *all* events that may occur at some time in the future. We assume that each event may be described in such a manner that it is possible to evaluate' at some future time the question of whether or not the event has occurred. This set of events in effect represents a state vector to define the "world" state of the system under observation. We may, in fact, explieitly define the state of this system as a binary message composed of one binary bit for each event, where the location (i.e., the i-th position in the message) corresponds to a particular event (i.e., the i-th event).` A zero bit in the i-th position will indicate that the event has not occurred and a one bit will indicate that is has occurred. At he present time the message contains all zeros since we are referring to evens that have not yet occurred.

We may further assume that there exists a set of prior probabilities (P_i) for the event set which indicates the likelihood of finding a one in each event position when we read the "message" at some specified future time. These probabilities are therefore an implicit function of the time interval which begins when we evaluate the values of the probabilities and ends when we plan to observe the content of the message.

As a result of the above conceptual model for the potential occurrence of events, we may write an expression for the information we know at the beginning of the time interval with respect to the content of our world message at the end of the time interval:

$$I = + \sum_i [P_i \ln P_i + (1 - P_i) \ln (1 - P_i)]. \tag{34}$$

The form of the above expression is based upon the fact that the event not occurring, as well as the observation that the event occurs, provides information. This expression has a minimum when all the P's are equal to one - half, corresponding to a completely random chance of occurrence of the events over the specified time interval—in other words, a complete-lack of knowledge (i.e., a neutral position) about the likelihood of occurrence. The maximum occurs when all the P's are either zero or one which implies complete certainty as to the occurrence or nonoccurrence of the events.

The basic goal of the cross impact technique is to set up a measuring system whereby an individual's knowledge concerning a set of events can be quantified for the purpose of making a meaningful comparison among a set of individual estimates and collating the estimates into a group assessment. There are two aspects of this information or knowledge which are explicitly sought:

1. The prior probabilities of the events occurring given the world as the individual views it at the time.
2. The causal relationships, if any, whereby events may influence the occurrence or nonoccurrence of others.

In order to obtain a measure of this second phenomenon, we now take an approach analogous to a weakly interacting subsystems assumption in statistical mechanics.

For a set of N events, there are 2^N different outcomes in terms of the total message containing zero or one for each event. If the events are independent, the probability of receiving any particular message is

$$\prod_{l \in S} P_l \prod_{m \notin S} (1 - P_m), \tag{35}$$

where the index 1 ranges over, those events which occur (subset S) in the message and m ranges over those events which do not occur (not in S). The sum of these probabilities over all the 2^N messages or possible world states is one.

Since our events are not necessarily independent and certain messages may be more or less likely than the quantity implied in Eq. (35), we introduce a set of statistical weights (W_k)[8] and define the probability of obtaining the k-th outcome of the 2^N as

$$\mathbb{P}(k) = W_k \prod_{l \in S} P_l \prod_{m \notin S} (1 - P_m). \tag{36}$$

If we could specify the actual physical interaction process between these events, then the W_k's would be obtainable from the analytical model of the process as is typically done in statistical mechanics. In our case they have to be viewed as quantities which can usually only be obtained by subjective estimates. It is still true, however, that the $\mathbb{P}(k)$'s must satisfy

$$\sum_{k=1}^{2^N} \mathbb{P}(k) = 1. \tag{37}$$

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[8] Formally these weights may be viewed as made up of a complex expression of conditional probabilities.

We now rewrite Eq. (37) utilizing (36) and a new set of 2^N constants (C's) as:

$$C^0 + \sum_{j=1}^{N} C_j^1 P_j + \sum_i \sum_{j>i} C_{ij}^2 P_i P_j + \sum_i \sum_{j>i} \sum_{k>j} C_{ijk}^3 P_i P_j P_k + \dots$$
$$\dots + C_{1,2,3\dots N} P_1 P_2 \dots P_N = 1. \quad (38)$$

Each of the C's in the above expression is uniquely defined as a linear combination of the W's in Eq. (36).

We now view Eq. (38) as constraint upon maximizing Eq. (34) for *the* total informa tion. Using the Lagrange approach we then have, for *any* particular event i (taking the differential with respect to P_i):

$$\ln \frac{P_i}{1-P_i} = \lambda \Big[C_i^1 + \sum_{j\neq i} (C_{ij}^2 + C_{ji}^2) P_j + \sum_{j\neq i} \sum_{k\neq i,j} (C_{ijk}^3 + C_{jik}^3 + C_{kji}^3) P_j P_k$$
$$+ \text{Higher Order in } P\text{'s} \Big]. \quad (39)$$

Note that the right hand side' of the above equation does not contain P_i.

It now becomes clear what sort of approximations are being made in the cross impact relation obtained earlier. In order to reduce Eq. (39) to the earlier result; e.g., Eq. (13), we do the *following:*

1. For any reasonable event set, it is infeasible to expect an individual to answer 2^N question in order to evaluate all the C's or W's. Therefore, we in effect ignore terms of P^2 or greater, hoping that the three- or four-way interactions are small.
2. The derivation is valid for the set of all potential events. Usually only a specific subset with *a* range of about *5* to 100 events is utilized. Therefore all events not specified in the application of the cross impact analysis are in effect lumped into the constants, since their prior probability of occurrence is assumed constant within the scope of the estimation process.

Under these approximations we may rewrite (39) as

$$\ln[P_i/(1-P_i)] = \gamma_i + \sum_{j\neq i} C_{ij} P_j, \quad (40)$$

where the Lagrange multiplier ? has been incorporated *in* the constants and where both $?_i$ and C_{ij} are a function of the events not specified. In essence, the ?-coefficient may be viewed as a measure of the "temperature" of the environment created for the i-th event by the unspecified set of events.

The ratio

$$C_{ij}^* = \frac{C_{ij}}{\gamma_i} \quad (41)$$

gives a good measure or indication of how sensitive the i-th event is to the j-th event as compared with the rest of the environment_. Note that if $?_i$ is positive the unspecified events contributed to the occurrence of the i-th event and vice versa. Also if C_{ij} is positive, then the j-th event contributed the occurrence of the i-th event.

The effect of ignoring higher order interactions among specified events can be measured by asking a subjective question of the form:

> Given the most favorable (or unfavorable) set of circumstances for event i with respect to the occurrence or nonoccurrence the remaining specified events, what is your estimate *of* the resulting probability for the occurrence of the i-th event?

This allows one to calculate two other values for $?_i$ in addition to the one initially obtained. The range of $?_i$ defined by the difference of these two values in principle measures the inaccuracy in the approximation due to ignoring the higher order terms in the specified P's

The three values of ? are defined by

$$\gamma_i = \phi(P_i) - \sum_{j \neq i} C_{ij} P_{j(\text{original})}, \tag{42}$$

where P_i is the original estimate

$$\gamma_i^1 = \phi(P_i^f) - \sum_{j \in C_{ij} > 0} C_{ij}, \tag{43}$$

where P_i^f is the favorable estimate

$$\gamma_i^2 = \phi(P_i^u) - \sum_{j \in C_{ij} < 0} C_{ij}, \tag{44}$$

where P_i^u is the unfavorable estimate.

For calculating $?_i^1$ we have assumed $P_j = 0$ if $C_{ij} < 0$ and $P_j = 1$ if $C_{ij} > 0$. The converse is assumed for calculating $?_i^2$. The explicit measure of the inaccuracy in ignoring the higher order terms would then be

$$\frac{\gamma_i^1 - \gamma_i^2}{\gamma_i}. \tag{45}$$

If one does choose to obtain values for $?_i^1$ and $?_i^2$ an interpolation procedure may be established to modify $?_i$ such that P_i will range between P_i^u and P_i^f as the other P's are allowed to vary in order to examine different potential outcomes for the set of events. Therefore, the effect of higher order "interactions" among the event set can at least be approximated.

This particular view of the cross impact leads one to the conclusion that two types of events should be specified in any cross impact exercise:

Dependent Events: Those whose occurrence are a function of other events in the set.

Independent Events: Those whose occurrence are largely unaffected by the other events in the set but may influence some subset of the other events.

These events may be obvious at the initial specification of the event set (e.g., the occurrence of a natural disaster) or they may be determined empirically when

$$\left|\frac{C_{ij}}{\gamma_i}\right| \ll 1, \text{ for all } j. \tag{46}$$

If the event is independent, then there is no need to ask for information on the impact of the other events on it. This has the benefit of reducing the estimation effort on the part of the respondents to the exercise.

While we can, therefore, obtain some idea of the significance of the unspecified events in terms of their impact on the specified set of events, there is no analytical guidance for resolving the fundamental question of what particular events should make up the specified set. This procedure is entirely dependent upon the group which will be supplying the estimates and the general problem area that is to be examined. However, the author does feel that the concept of *Dependent and Independent Events* should be introduced at the stage of actually formulating the event set.

Given an on-line computer system for collecting cross impact estimates, there is, in principle, no hindrance to extending the approach developed in this paper to allow estimators to express three way or higher order interactions when they think they are significant. Equation (39) may be used to specify, the higher order cross impact factors. Also, the pairwise interactions can be evaluated and-specific higher order questions can be generated about those pairwise interactions Which appear to be crucial or dominant. However, extensions of this sort are feasible only with groups that will make regular use of such techniques and which have `had some degree of practice with similar quantitative approaches:

Example

The following goes step by step through a cross impact exercise set up in an online user mode on a computer. The numeric quantities reflect the inputs of a young economist who felt that the behavior of the resulting model reflected his judgment. It took him three iterations (in terms of changes to the "conditional" probabilities) to arrive at this situation. For the sake of brevity the final inputs are presented as if they all occurred on the first iteration. The program also operated in a long or short explanation mode according to the users' option and did supply a verbal definition of probability ranges as well as an odd to probability conversion table. The first thing the user sees, if he wishes, is a list of the events. It is, however, not necessary to store the events themselves as they are referenced individually by a number throughout the exercise. All the user needs is a hard copy list, which indicates the event number for each event statement. This is particularly useful where confidentiality of the events under consideration is of importance. The long form (i.e., full explanation) of the interaction is presented.

CONSIDER THE FOLLOWING EVENTS AND THE POSSIBILITY OF THEIR OCCURRENCE BETWEEN NOW AND THE YEAR 1980:

1. THE U.S. GETS IN A TRADE WAR WITH ONE OR MORE OF ITS MAJOR TRADING PARTNERS (JAPAN, CANADA, WESTERN EUROPEAN COUNTRIES).
2. COMPREHENSIVE TAX REVISION S ENACTED WITH MOST PRESENTEXEMPTIONS AND EXCLUSIONS REMOVED, BUT WITH RATES LOWERED.
3. RIGOROUS ANTI-POLLUTION STANDARDS ARE ADOPTED AND STRICTLY ENFORCED FOR BOTH AIR AND WATER.
4. THE U.S. AVERAGES AT LEAST 4 PERCENT PER YEAR GROWTH RATE OF REAL GNP FOR THE TIME FRAME THROUGH 1980.

5. DEFENSE SPENDING DECLINES STEADILY AS A PERCENT OF THE FEDERAL GOVERNMENT'S ADMINISTRATIVE BUDGET.
6. THE U.S. EXPERIENCES AT LEAST ONE MAJOR RECESSION (GNP DECLINE IS GREATER THAN 5 PERCENT FOR A DURATION GREATER THAN 2 QUARTERS) DURING THE TEN-YEAR PERIOD.
7. A FEDERAL INCOME MAINTENANCE SYSTEM (E.G., NEGATIVE INCOME TAX) REPLACES ESSENTIALLY ALL CURRENT STATE AND LOCAL WELFARE PROGRAMS.
8. THE OIL IMPORT QUOTA SYSTEM IS PHASED OUT AND DOMESTIC OIL PRICES ALLOWED TO FALL TO THE WORLD PRICE.
9. THE U.S. AGRICULTURAL PRICE SUPPORT SYSTEM IS DISMANTLED.
10. A FEDERAL-STATE AND LOCAL REVENUE-SHARING PROGRAM IS ADOPTED WHICH ALLOCATES AT LEAST 5 PERCENT OF FEDERAL REVENUES TO STATE AND LOCAL GOVERNMENTS.

IN THIS EXERCISE WE WILL BE ASKING FOR YOUR SUBJECTIVE PROBABILITY ESTIMATES. FOR THOSE OF YOU WHO WISH TO THINK IN TERMS OF ODDS, THE FOLLOWING CONVERSION EQUATION AND EXAMPLES MAY BE OF USE:

ODDS = A: B	EQUIVALENT TO PROBABILITY = A/(A + B)				
EXAMPLES	1:99 = .01	1:9 = .1	1:4 = .2	3:7 = .3	1:2 = .33
	2:3 = .4	1:1 = .5	3:2 = .6	2:1 = .66	7:3 = .7
	4:1 = .8	9:1 = .9	99:1 = .99		

A SEMANTIC EQUIVALENT TO THE NUMERIC PROBABILITIES MAY BE TAKEN AS:

VERY PROBABLE	>= .75
PROBABLE	> .5 BUT < .75
EITHER WAY	= .5
IMPROBABLE	< .5 BUT > .25
VERY IMPROBABLE	< .25

TEAR OFF THE ABOVE LIST OF EVENTS FOR REFERENCE BY EVENT NUMBER THROUGHOUT THE REST OF THIS EXERCISE.

STEP 1: OVERALL PROBABILITIES

PLEASE SUPPLY YOUR BEST ESTIMATE FOR THE PROBABILITY THAT EACH OF THE EVENTS WILL OCCUR AT SOME TIME BETWEEN NOW AND 1980.

UNLESS YOU CHANGE THEM ALL, THE PROBABILITIES ARE INITIALLY SET TO .5 WHICH IS EQUIVALENTTO EXPRESSING A NO JUDGMENT FOR THE PARTICULAR EVENT WITH RESPECT TO THE ABOVE QUESTION.

ESTIMATES: 2, .3, 3, .6, 5, .4, 6, .3, 7, .6, 8, .2, 9, .1, 10, .6

SUMMARY STEP 1

EVENT	1	2	3	4	5	6	7	8	9	10
P =	.50	.30	.60	.50	.40	.30	.60	.20	.10	.60

IF SATISFIED HIT RETURN KEY, IF NOT TYPE SOMETHING FIRST.

STEP 2: CONDITIONAL PROBABILITIES

IN THIS STEP YOU ARE ASKED TO ASSUME FOR THE PURPOSE OF ANALYSES THAT YOU HAVE BEEN PROVIDED CERTAIN KNOWLEDGE AS TO WHETHER A PARTICULAR EVENT WILL OR WILL NOT OCCUR IN THE STATED TIME FRAME. BASED UPON THIS HYPOTHETICAL SITUATION, FOR EACH EVENT IN TURN, PLEASE INDICATE ANY RESULTING NEW ESTIMATE FOR THE PROBABILITY OF OCCURRENCE OF THE OTHER EVENTS.

UNLESS YOU CHANGE THEM, THESE CONDITIONAL PROBABILITIES ARE SET EQUAL TO THE OVERALL PROBABILITIES.

ASSUME EVENT 1 IS CERTAIN TO OCCUR. INDICATE YOUR ESTIMATES OF CHANGES IN THE PROBABILITIES OF OCCURRENCE FOR THE OTHER EVENTS.

ESTIMATES: 2, .25, 3, .55, 4, .4, 5, .3, 6, .4, 7, .55, 8, .1, 9, .05, 10, .55

At this point the computer calculates each C_{ij} from Eq. (19); or if the event had been assumed to not occur, Eq. (21) would have been used. If no change had been indicated, the corresponding C would be set to zero.

The computer informs the user about the occurrence or non-occurrence of an event according to how he specified the overall probabilities. If he specifies a probability of .5 or less, he is told to assume the event occurred; if more than .5, than he is told to assume it did not occur. This policy is arbitrary. In this example the user was told to assume occurrence for events 1, 2, 4, 5, 6, 8, and 9 and to assume non-occurrence for events 3, 7, and 10.

The user is allowed only two digit specification of a probability which must lie between (and including) .01 and .99. If he enters a zero or one, it is automatically changed to .01 or .99 respectively.

When the user has gone through all the events in the above manner and is satisfied with his inputs, then the $?_i$'s are calculated from Eq. (42).

The user is now presented a summary of his inputs and the converse "conditional" probability to the one supplied which is calculated from Eq. (22).

SUMMARY CONDITIONAL PROBABILITIES BASED UPON OCCURRENCE AND THEN NONOCCURRENCE, NC INDICATES NO CHANGE FROM OVERALL P

E	1	2	3	4	5	6	7	8	9	10
1	OV PR	.45	NC	.40	.45	.75	NC	.45	.45	NC
1	.50	.52	NC	.60	.53	.38	NC	.51	.51	NC
2	.25	OV PR	.28	.35	NC	.20	.38	.35	.35	.34
2	.36	.30	.33	.25	NC	.35	.20	.29	.29	.25
3	.55	.65	OV PR	.65	.70	.50	NC	.65	.65	.66
3	.65	.58	.60	.55	.53	.64	NC	.59	.59	.50
4	.40	.60	.51	OV PR	.55	.30	.53	.55	.55	.53
4	.60	.46	.49	.50	.47	.59	.45	.49	.49	.45
5	.30	.50	.39	.50	OV PR	.35	.47	NC	NC	.43
5	.51	.36	.42	.31	.40	.42	.30	NC	NC	.35
6	.40	.25	NC	.10	.25	OV PR	.27	.25	.25	.27
6	.22	.32	NC	.62	.34	.30	.35	.31	.31	.35
7	.55	.75	NC	.70	.75	.55	OV PR	NC	NC	.66
7	.65	.53	NC	.49	.49	.62	.60	NC	NC	.50
8	.10	.15	NC	.25	.25	.10	NC	OV PR	.30	.24
8	.36	.22	NC	.16	.17	.26	NC	.20	.19	.15
9	.05	NC	NC	.15	NC	.05	.15	.20	OV PR	.15
9	.19	NC	NC	.07	NC	.13	.05	.08	.10	.05
10	.55	.75	.59	.70	.75	.50	.66	NC	NC	OV PR
10	.65	.53	.62	.49	.49	.64	.50	NC	NC	.60

ASSUMING ALL THE OTHER EVENTS OCCUR OR DO NOT OCCUR SO AS TO ENHANCE THE GIVEN EVENT, THE MOST FAVORABLE PROBABILITY FOR EACH EVENT IS:

EVENT:	1	2	3	4	5	6	7	8	9	10
MFP =	.86	.73	.91	.89	.81	.85	.94	.78	.76	.95

ASSUMING ALL THE OTHER EVENTS OCCUR OR DO NOT OCCUR SO AS TO INHIBIT THE GIVEN EVENT THE LEAST FAVORABLE PROBABILITY IS

EVENT:	1	2	3	4	5	6	7	8	9	10
MFP =	.16	.06	.22	.11	.09	.02	.17	.01	.01	.13

FOLLOWING IS A TABLE OF THE RELATIVE CAUSAL WEIGHTS (CROSS IMPACT FACTORS) OF ONE EVENT (COLUMN) UPON ANOTHER (ROW) AND A MEASURE (GAMMA) OF THE EFFECT OF EVENTS NOT SPECIFIED APPEARS IN THE DIAGONAL ELEMENT. MINUS INDICATES AN INHIBITING EFFECT.

CROSS IMPACT TABLE

E	1	2	3	4	5	6	7	8	9	10
1	+.23G	−.29	0	−.81	−.33	1.57	0	−.25	−.22	0
2	−.50	−1.33G	−.23	.46	0	−.77	.90	.29	.25	.42
3	−.41	.31	−.30G	.43	.74	−.58	0	.27	.24	.68
4	−.81	.58	.07	−.05G	.33	−1.21	.33	.25	.22	.33
5	−.88	.58	−.14	.81	−1.02G	−.31	.74	0	0	.36
6	.88	−.36	0	−2.70	−.42	+.88G	−.38	−.31	−.28	−.38
7	−.41	.99	0	.88	1.16	−.29	−.91G	0	0	.68
8	−1.62	−.50	0	.58	.48	−1.16	0	−.97G	.60	.58
9	−1.49	0	0	.93	0	−1.07	1.25	1.01	−3.29G	1.25
10	−.41	.99	−.14	.88	1.16	−.58	.68	0	0	.74G

G INDICATE THE GAMMA FACTOR AS THE DIAGONAL ELEMENT.

The user may infer from the cross impact factors in the previous table the relative rank order with respect to the effect of one event upon another as interpreted from his judgments on the probabilities.

The next step is for the computer to present the user with a forecast of which events will occur. To do this it is assumed that the perception of the likelihood of the event occurring produces the causal effect, and not the actual time of occurrence. With this time independent view we can assume it is reasonable to apply a cascading perturbation approach to forecasting occurrence. This is done as follows:

(1) Examine the overall probabilities and determine which event or events is closest to zero or one.

(2) If the event is close to zero, assume it will not occur or if it is close to one assume it will occur (this is the smallest possible perturbation).

(3) Based on (2), calculate, new probabilities for the remaining events.

(4) Begin step 1 again for those events which have not already been assumed to occur or not occur.

The above sequence is repeated until the outcome is established for all events unless the final probability is .5, in which case no outcome is forecast.

The following is what happens for the above example where each row is one cycle of the above cascade iteration procedure. The user can observe how the probabilities are affected. Note that the initial estimates on events three, seven, and ten are reversed.

FORECASTED CERTAINTY SEQUENCE, THE + INDICATES OCCURRENCE AND THE - NON-OCCURRENCE

E	1	2	3	4	5	6	7	8	9	10
0	.50	.30	.60	.50	.40	.30	.60	.20	.10	.60
1	.51	.29	.59	.49	.40	.31	.60	.19	---	.60
2	.52	.28	.58	.48	.40	.32	.60	---	---	.60
3	.55	---	.55	.43	.35	.36	.52	---	---	.52
4	.61	---	.45	.36	---	.46	.37	---	---	.37
5	.72	---	.36	---	---	.76	.26	---	---	.26
6	.86	---	.27	---	---	+++	.21	---	---	.18
7	+++	---	.24	---	---	+++	.18	---	---	.16
8	+++	---	.22	---	---	+++	.17	---	---	---
9	+++	---	.22	---	---	+++	---	---	---	---
10	+++	---	---	---	---	+++	---	---	---	---

YOU MAY REPEAT THE SEQUENTIAL ANALYSES WITH NEW INITIAL PROBABILITIES. YES (1), NO (2), CHOICE?

SUMMARY OF PERTURBED OUTCOMES FOR CROSS IMPACT SET OF ECONOMIC EVENTS

THE + SIGN INDICATES WHICH EVENTS WERE FORECASTED TO OCCUR FOR THE GIVEN OVERALL PROBABILITIES	Overall Probability	Resulting Outcome	Change in Overall Probability	Resulting Outcome	Change in Overall Probability	Resulting Outcome
1. U.S. in trade war	.5	+				
2. Major tax revision	.3				.99	+
3. Rigorous anti-pollution standards	.6			+		+
4. GNP at 4% real growth	.5			+		+
5. Defense spending declines	.4		.8	+		+
6. "Recession" occurs	.3	+				
7. Income maintenance enacted	.6			+		+
8. Oil Import quotas eliminated	.2					+
9. Agricultural price support	.1					
10. Revenue sharing enacted	.6			+		+

The user may now examine the sensitivities of this model by choosing to modify one or more of the overall probabilities and holding the rest and the cross impact factors constant. This would correspond to assuming a basic change in policies effecting the likelihood of a particular event. In this instant the user chose to increase the probability that defense spending decreased and then to separately view the effect of a major tax revision. The effects of these choices are summarized and compared to the original result above.

If the user is not satisfied with the behavior of the model he has built up, he may go back and make changes to the original overall probabilities and/or the conditionals until he has obtained satisfactory behavior.

If the activity were part of a Delphi or other group exercise, then once a user was satisfied with his estimates they would be collected in order to obtain a group response.

The group response would be determined by a linear average of the cross impact factors and the gamma factors-not the probabilities. Then each individual would be able to see similar inferences as the above for the group view with the addition of a matrix which compared the number of individuals who estimated a positive, negative, or no impact relation' between each event combination. In the group case one would also have to allow the estimator to indicate which cross impacts he has a no judgment position on. The computer would than supply for him, if he wishes, the average supplied by the rest of the group for that particular cross impact relationship.

Applications

The intriguing aspect of the cross impact formalism is its utility to a rather broad range of applications. The first application is as an aid to or tool for an individual in organizing and evaluating his views on a complex problem. The structure offers the individual more freedom in expressing the event set than the constraints of mutual exclusiveness imposed in decision tree and table type approaches. There also appears to be some compatibility between the pair-wise examination of causal relationships and the way many individuals think about causal effects. This is true to the extent that crosses impact formalism maybe utilized quite easily by individuals without any formal training in decision theory or probability. The author has,' for example, gone through the creation and evaluation of a set of five events with a group of high school students within a one-hour period, using a computer terminal to perform the calculations. That particular exercise stimulated a great deal of class discussion as to under what economic conditions the students would plan to have children. The educational utility of the cross impact formalism, as well as other Delphi-oriented communication structures, has largely gone unnoticed.

The main problem encountered in utilizing the technique is that some individuals are so accustomed to the Bayes theorem that they will habitually apply it in responding to the Cross impact questions.

Once some,, members in an. organization have begun to employ the approach for their individual benefit then it becomes quite easy to introduce it as a communication form for expressing quite precisely to others in the group how they view the causal relationships involved in the problem under consideration. The benefit here is in allowing the group to quickly realize where disagreement exists in both the direction, as well as relative magnitude, of the impacts. This can eliminate a lot of superfluous discussion about areas of agreement.

Whether the evaluation of an event set is carried out in a committee, conference, Delphi, or some combination, of these processes, it is mandatory that the group involved reach agreement and understanding on the specification and wording of the event set. In addition, the actual cross impact exercise may cause the group to desire modification of the event set.

In utilizing the technique for serious problems, there would appear to be benefits for groups of both decision makers and analysts. In addition, it may solve a problem that now exists in attempting to set up efficient communication structures between these two groups. The analyst attempting to build simulations or models of complex processes of interest to the decision maker very often encounters causal relationships dependent upon policy and decision options that defy any reasonable attempt at incorporation into the model, except in the form of prejudging the outcome of policy or decision options. At times these choices are so numerous that they are effectively buried and become hidden assumptions in; the logic of complex simulations. The cross impact technique offers he analyst an opportunity, to leave portions of the simulation logic arbitrary; thus, the users of the simulation may utilize a cross impact exercise to structure the logic of the simulation when they wish. While this application has not yet been demonstrated, it may turn out to be a major use of the cross impact technique. There is considerable advantage to be had from introducing a greater degree of flexibility in the application of the more comprehensive simulations being built to analyze various organizational, urban, and national problems.

As with many Delphi structures[9], it is quite feasible to design an on-line conference version of the cross impact exercise which would eliminate delays in processing the group results and allow the conferees to modify their views at will. It would be necessary to tie this particular conference structure to a general discussion conference (such as the "Delphi Conferencing" system) in order that the group can first specify the event s et and later discuss disagreement on causal effects.

If one considers the basic functions performed in the planning operation of organizations, whether they be corporate or governmental, there are two other types of conference structures that should be added to the general discussion format and the cross impact conference structure.

One is a resource allocation conference structure which allows a group to reach agreement on what is the most suitable allocation of the organizational resources to bring about the occurrence of the type of event which the organization controls or influences (i.e., controllable events). Various program options evaluated in terms of resources required and probability of accomplishment as a function of time and resource variability would evolve from this type of conference.

The other type of conference structure involves forecasting the environment in which the organization must function. This conference would be used to generate

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[9] See "Delphi Conferencing" hy Murray Turoff, *Technological Forecasting and Social Change 3*, No. 2, 1971. Also, "The Delphi Conference," in *The Futurist,* April 1971, provides a summary report.

information on the uncontrollable events which specify the environment and or their, likely occurrence over time.

The resource allocation conference may use various optimization techniques, such as linear programming, to aid the group members in their judgments. The environmental forecasting conference may: use such tools as trend, correlation, or substitution analysis routines to aid the conference group.

The cross impact conference structure may now be viewed as a mechanism for relating uncontrollable events expressing potential environmental situations to controllable events expressing organizational options. The general discussion conference allows the group or groups involved to maintain consistency and resolve disagreements.

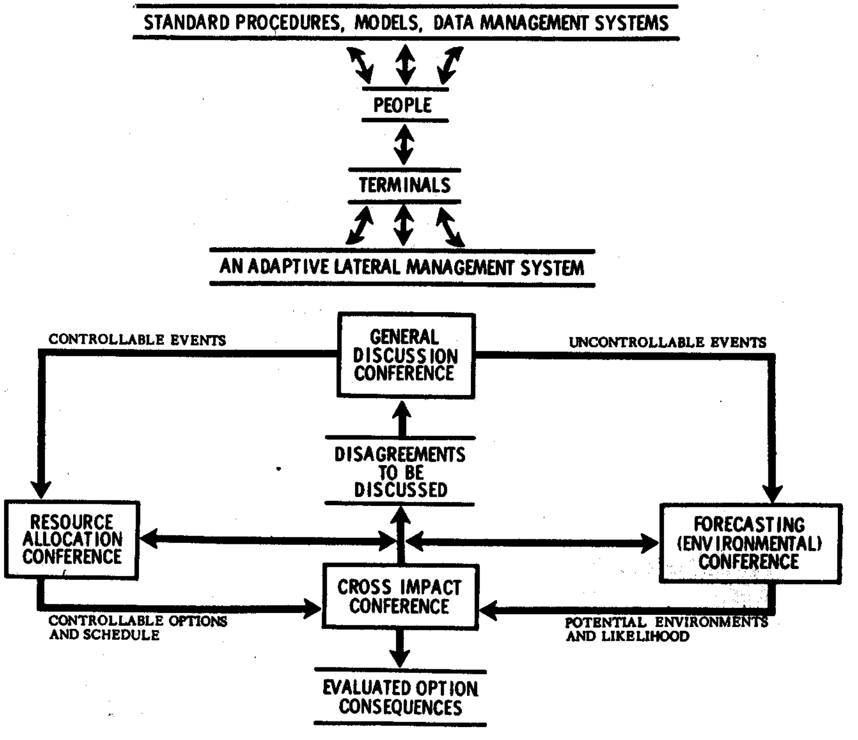
Initial design formats for all these conference structures already exist to some extent in the various paper and pencil Delphi's that have been conducted to date. It remains for some organization to piece these together within the context of a modern terminal oriented computer-communications system, Given, such a system; represented in the accompanying diagram, an organization faced with a specific problem may first, and quickly, bring: together the concerned group via the terminals and the general discussion conference format to; arrive at specifications for the resource allocation and forecasting conferences. These two latter conferences may involve only subsets of the total group and may draw on added expertise as needed. Using the cross impact to correlate the results of the other two efforts, the variability of options versus potential environments can be examined. The sought-after, result is a set of evaluated options suitable for providing an analysis basis for a decision.

One may envision simultaneous replication of this four-way conference structure focusing on different problems which may also relate to different levels of concern within the organization, A set of procedures could also be introduced for moving the results of one problem analysis to a higher level conference group or for sending requests to resolve particular uncertainties down to a conference group at a lower level.

The main advantage of such a system is the organization's ability to draw upon the talent needed for the problem on a timely and efficient basis, regardless of where it resides with respect to either geography or organizational structure. Also inherent in this type of system is the view that the individuals in an organization are the best vehicle for filtering the information appropriate to a particular problem out of established data management system and other constant-type organizational procedures. The mistake often made by designers of management information systems is the assumption that there is a standard algorithm which will continually transform the normal flow of organizational data into a form suitable for management purpose.[10] This is only true when the organization is faced with an unchanging environment, and very few organizations, unless they are deluding themselves, can claim that view in this day and age.

The author views a Management Information System as just this four-way conference structure existing in a design scheme which allows groups to easily shift from one format to another and which may be replicated either to improve lateral

[10] This point is developed further in "Delphi and Its Potential Impact on Information Systems," by Murray Turoff, in the Proceedings of the Fall Joint Computer Conference, 1971.



communication at various organizational levels or to tackle a multitude of problems. The concept is basically a lateral communication system and presupposes an organizational environment which supports or fosters lateral communication. It is also highly adaptive and able to respond to a changing environment which in turn may impose changing requirements on the organization. Many large organizations already have fairly extensive planning and forecasting efforts scattered through their various divisional or vertical organizational structures. It is less clear r that these numerous segments of the organization can effectively relate to one another and the organization as a whole. Current methods of doing so, involving frequent travel and extended meetings, are often prohibitive on a time and effort basis. However, given the requirements facing organizations today, the growing availability of terminals, computer hardware and software to support conferencing, and the availability of digital communication networks providing reasonable communication, costs, it can be expected that the system of the type described here will come into being over this next decade.

Acknowledgment

As one may suspect, this paper has evolved out of a number of earlier drafts. I would like to thank the following individuals for their aid in terms of comments and reviews (both pro and con) : Dr. Ronald Bass, Dr. Norman Dalkey, Mr. Selwyn Enzer, Dr. Felix Ginsberg, Professor Jack Goldstein, Mrs. Nancy Goldstein, Professor Robert Piccirelli, Miss Christine Ralph, Professor Richard Rochberg, Dr. Evan Walker, Dr. John Warfield, and Mr. Dave Vance.

Annotated Bibliography

The initial paper on cross impact was published in 1968 by Gordon and Hayward. Other. papers specifically on cross impact are those by Dalby, Enzer, Johnson, and Rochberg. Very closely related to the cross impact formalism is the cross support formalism in C. Ralph's work. This formalism in essence makes a clear distinction between dependent events, which are defined as goals, and independent events, which are related, to resource allocation choices. Also related to the cross impact problem is, the management matrix formalism referenced and discussed in the book by Farmer and Richman and in a paper by Richman. The measures of association concept in business, problems (see Perry's paper for a review) is another variation of the same problem. The formal problem of defining measures of association or correlation coefficients are discussed in articles by Costner and Goodman.

The use of the logistic type equation in statistics for cases in which one is concerned with a binary outcome type process is reviewed quite well in the papers by Cox and Neter. The use of the logistic equation in economics (logic regression) is found in the works of Sprent and Theil. The Fermi-Dirac distribution in physics is discussed in Born and at a more advanced level in Tolman. The use of the logistic distribution in Technological Forecasting as a modeling tool is discussed in Ayres' book. The mathematical properties of the logistic equation and its usefulness to model population growth is reviewed in the book by Davis.

The point that all prior probabilities are conditional is brought out quite clearly in the book by Savage. Raiffa's book contains an excellent guide to the philosophical differences that surround the concept of subjective probability and inference. Tribus' book is one of the few works that treats the "weight of evidence" measure (e.g., defined in the paper as the occurrence ratio) in some detail and references earlier papers on this topic. The discussion of unsettled problems of probability theory in Nagel's book is also relevant.

Two papers by Ward Edwards appearing in one book edited by Kleinmuntz and one book edited by Edwards and Tversky review the psychological experiments to determine if humans make judgments on a Bayesian basis. Edwards asserts, on the basis of his work, that humans are conservative; i.e., always making more conservative estimates than would be implied by the use of Bayes theorem. A more recent experiment by Kahneman and Tversky appears to indicate that man "is not Bayesian at all." These authors propose a "representativeness" heuristic; wherein, "the likelihood that a particular 12-year old boy will become a scientist, for example, may be evaluated by the degree to which the role of a scientist is representative of our image of the boy." This view does not appear to be too far removed from the "causality" view adopted in approach of this paper to the cross impact problem. The average person deals almost everyday with at least a subconscious' process for estimating non-recurrent and transient events. The

Bayesian approach to modeling this subjective probability process does not appear to fit or explain the judgments made. However, the experiments to date do appear to confirm that some sort of universal or consistent model exists which humans of very different backgrounds and training are in fact using.

The paper by DeWitt, which deals with the philosophical problem of inferring reality from quantum mechanics, is an excellent review of what the author feels are analogous difficulties with justifying cross impact. The chapter in Bohm's book, conjecturing that the human mind may function with a quantum mechanical type thought process, may, to a limited degree, be viewed as circumstantial support for the propositions developed in this paper. If Bohm were correct, it should not be a complete surprise that a macro statistical quantum mechanical distribution (Fermi-Dirac) can be used to correlate measurements of subjective estimates by a group of humans. Walker's paper also explores the potential relationship of quantum mechanics to the nature of consciousness. Bohr also argues in his writings for the more universal application of quantum mechanics to the human thought process.

Also, Reichenbach in his book examines the relationships between quantum mechanics and the calculus of probabilities. In particular, Reichenbach interprets posterior probabilities as those resulting from measurements, and priors or "potential" as those arising from the physical theory. He further discusses the need for a three-value logic system to deal with "causal anomalies"-true, false and indeterminacy. The threevalued-logic proposition of Reichenbach leads to the speculation that if there is a rigorous foundation for the theory of cross impact, it may lie in the work taking place in "fuzzy" set theory (i.e., a class which admits of the possibility of partial membership in it is called a fuzzy set). In cross impact the set of all possible events may be considered as made up of two subsets, those that will occur and those that will not. Any particular event may have partial membership in either set. That which we have termed the probability of occurrence is referred to by the "fuzzy" set theory people as the membership function (i.e., see Zadeh).

Umpleby's work represents a first and interesting attempt to tie the cross impact formalism to the resource allocation problem in at least an automated game mode. The problem of how to average probability estimates among a group is crucial to utilizing other cross impact systems. This is reviewed in Brown's paper and in Raiffa's book. The methodology proposed here at least explicitly avoids the question by averaging, for the group, correlation coefficients having a plus to minus infinity range.

In addition to the published literature there are at least three alternative methods under consideration or in use. These additional methods are proposed by Dalkey, or RAND, Enzer, of the Institute for the Future, and Kenneth Craver, of Monsanto. Extensive modifications to the original treatment by Gordon have recently been proposed by Folk, of the Educational Policy Research Center at Syracuse, and by Shimada, of Hitachi Ltd., Japan. Also the current work in the area of "Relevance Trees" represents attempts to tackle the same class of problems by unfolding the matrix structure into a tree structure. The concept of multi-dimensional scaling m psychology is also related to the cross impact problem (see J. D. Carroll's paper).†

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† The Carroll-Wish paper is in the next chapter.

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